

J. Piekarewicz

Department of Physics, Florida State University, Tallahassee, FL 32306

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We calculate the longitudinal response of ^{208}Pb using a relativistic random-phase approximation to three different parameterizations of the Walecka model with scalar self-interactions. From a nonspectral calculation of the response—that automatically includes the mixing between positive- and negative-energy states—we extract the distribution of strength for the isoscalar monopole, dipole, and high-energy octupole resonances. We employ a consistent formalism that uses the same interaction in the calculation of the ground state as in the calculation of the response. As a result, the conservation of the vector current is strictly maintained throughout the calculation. Further, at small momentum transfers the spurious dipole strength—associated with the uniform translation of the center-of-mass—gets shifted to zero excitation energy and is cleanly separated from the sole remaining physical fragment located at an excitation energy of about 24 MeV; no additional dipole strength is observed. The best description of the collective modes is obtained using a “soft” parameterization having a compression modulus of $K = 224$ MeV.

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Almost forty years ago Thouless wrote a seminal paper on vibrational states in nuclei in the random-phase approximation [1]. There he showed how spurious states—such as those associated with a uniform translation of the center-of-mass—separate out cleanly from the physical modes by having their strength shifted to zero excitation energy. Thirty years later Dawson and Furnstahl generalized Thouless’ result to the relativistic domain placing particular emphasis on the role of consistency [2]. They showed how a fully self-consistent approach guarantees the conservation of the vector current as well as the decoupling of the spurious component of the isoscalar dipole ($J^\pi = 1^-; T = 0$) mode from the physical spectrum. These results emerged after a careful treatment of the negative-energy states. Indeed, neglecting their contribution resulted in a violation of the vector current as well as the appearance of substantial spurious strength in the response. These fundamental results emphasize that the Dirac single-particle basis is complete only when positive- and negative-energy states are included.

Relativistic models of nuclear structure have evolved considerably since they were first introduced by Walecka [3] and later extended by Serot [4]. Although the qualitative success of these models relies almost exclusively on the dynamics generated by the the scalar (σ) and vector (ω) mesons, several improvements have been introduced in order to enhance their quantitative standing [5,6]. Chief among them is the incorporation of scalar self-interactions which introduce important non-linearities into the equations of motion. Perhaps the greatest impact of these non-linear terms has been seen in the compression modulus of nuclear matter. In a linear model the compression modulus is predicted to be unreasonably large at $K = 547$ MeV. Yet this value can be reduced to $K = 224$ MeV by the mere inclusion of non-linear terms. We will show here how this “soft” parameterization yields excitation energies for various com-

pressional modes in fair agreement with experiment.

While scalar self-interactions are now incorporated routinely into most relativistic calculations of the nuclear ground state, their role on the dynamics of the excited states is just being unraveled; most calculations of the response of the mean-field ground state still use the linear model. Applying non-linear models becomes technically more difficult because the scalar-meson propagator no longer has a simple Yukawa form. At present the only calculations of the response that have incorporated non-linear terms are those by Ma and collaborators [7,8]. One of the main conclusions of their work is that “*a large discrepancy remains between theory and experiment in the case of the dipole compression mode*”. We now show that if one includes the full momentum dependence of the longitudinal response, a unique physical fragment emerges at low momentum transfer. This fragment—located at an excitation energy of $E \approx 24$ MeV—is identified as the isoscalar giant dipole resonance (ISGDR).

We start from a Lagrangian having an isodoublet nucleon field (ψ) interacting via the exchange of isoscalar sigma (ϕ) and omega (V^μ) mesons, an isovector rho (b^μ) meson, and the photon (A^μ). That is, the interacting Lagrangian density becomes [5,6]

$$\begin{aligned} \mathcal{L}_{\text{int}} = & g_s \bar{\psi} \psi \phi - g_v \bar{\psi} \gamma^\mu \psi V_\mu - \frac{1}{2} g_\rho \bar{\psi} \gamma^\mu \tau_a \psi b_\mu^a \\ & - \frac{1}{2} (1 + \tau_3) e \bar{\psi} \gamma^\mu \psi A_\mu - U(\phi). \end{aligned} \quad (1)$$

In addition to meson-nucleon interactions the Lagrangian density includes scalar self-interactions of the form

$$U(\phi) = \frac{1}{3!} \kappa \phi^3 + \frac{1}{4!} \lambda \phi^4. \quad (2)$$

Our theoretical program in the linear model has been described in great detail in several references [9,10]. Here we merely highlight the main features of the approach.

The longitudinal response of the mean-field ground state is defined by

$$S_L(\mathbf{q}, \omega) = \sum_n \left| \langle \Psi_n | \hat{\rho}(\mathbf{q}) | \Psi_0 \rangle \right|^2 \delta(\omega - \omega_n) \\ = -\frac{1}{\pi} \mathcal{I}_m \Pi^{00}(\mathbf{q}, \mathbf{q}, \omega), \quad (3)$$

where $\hat{\rho}(\mathbf{q})$ is the Fourier transform of the isoscalar vector density, Ψ_0 is the exact nuclear ground state, and Ψ_n is an excited state with excitation energy ω_n . Note that the response is directly related to the timelike polarization insertion Π^{00} . To compute the linear response of the ground state of spherical nuclei—such as ^{208}Pb —one starts by calculating ground-state properties in a mean-field approximation. In this mean-field theory (MFT) nucleons interact with the self-consistent field generated by all positive-energy nucleons; vacuum loops are neglected in this approximation. Such a calculation yields single-particle energies and wave-functions for the occupied states as well as the mean-field potential $\Sigma_{\text{MF}}(x)$. It is precisely this mean-field potential that one uses to compute the single-nucleon propagator nonspectrally:

$$(\omega \gamma^0 + i\gamma \cdot \nabla - M - \Sigma_{\text{MF}}(x)) G_F(\mathbf{x}, \mathbf{y}; \omega) = \delta(\mathbf{x} - \mathbf{y}). \quad (4)$$

There are several advantages in using a nonspectral representation for the nucleon propagator [9,10,12]. First, one avoids the artificial cutoffs and truncations that plague the spectral approach [2]. Second, both positive and negative-energy continua are treated exactly. As a result, the contributions from the negative-energy states to the response are included automatically. Finally, a nonspectral evaluation of G_F poses no more challenges, nor requires much more computational effort, than the corresponding calculation of an individual single-particle state. Having determined the occupied bound-state orbitals and the nucleon propagator, the evaluation of the uncorrelated—or single-particle—polarization becomes relatively straightforward [9,10].

To go beyond the simple single-particle response one must invoke the relativistic random-phase approximation (RPA). The RPA builds long-range coherence among the many particle-hole excitations with the same quantum numbers by iterating the uncorrelated polarization to infinite order [11]. Yet before going any further in the description of the RPA we must stress two issues of paramount importance. The first is consistency, which demands that the residual particle-hole interaction used in the RPA be identical to the interaction used to generate the mean-field ground state. Second, the consistent relativistic response of the mean-field ground state involves, in addition to the familiar particle-hole excitation, the mixing of positive- and negative-energy states. These new configurations are essential for the conservation of the vector current and for the removal of spurious dipole

strength from the physical spectrum. Although in the MFT it is consistent to neglect vacuum polarization [2], the mixing between positive- and negative-energy states remains of utmost importance.

The one new ingredient that we wish to add to our formalism is scalar self-interactions. The added complication arises from the fact that the scalar-mediated interaction no longer has a simple Yukawa form. Rather, the scalar propagator now satisfies a Klein-Gordon equation:

$$(\omega^2 + \nabla^2 - m_s^2 - U''(\phi)) \Delta(\mathbf{x}, \mathbf{y}; \omega) = \delta(\mathbf{x} - \mathbf{y}). \quad (5)$$

In infinite nuclear matter the scalar self-interactions introduce a trivial modification: the scalar meson now propagates with an effective mass $m_s^{*2} = m_s^2 + U''(\phi_0)$, rather than with its free-space value. In the finite system solving for the scalar propagator becomes technically more difficult, but not more than solving for the nucleon propagator of Eq. (4). We have computed the scalar propagator in momentum space and have expanded it in terms of spherical harmonics so that the angular integrals appearing in the RPA equations may be done analytically. A publication containing a more detailed description of our techniques will be forthcoming.

The benchmark by which every theoretical calculation of the nuclear response should be measured is the isoscalar giant dipole resonance. This is because the conservation of the vector current and the shift of spurious strength to zero excitation energy can only happen in a consistent calculation of the response. In Fig. 1 we display the distribution of isoscalar dipole strength in ^{208}Pb at the small momentum transfer of $q = 46 \text{ MeV}$ (or $q = 0.23 \text{ fm}^{-1}$) using parameter set NLC from Table I.

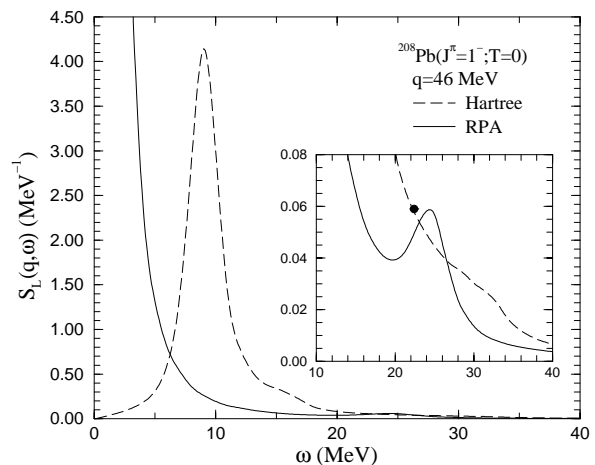


FIG. 1. Distribution of isoscalar dipole strength in ^{208}Pb at a momentum transfer of $q = 46 \text{ MeV}$. Calculations were done using parameter set NLC [6] while the experimental value (filled circle on inset) is from Ref. [14].

Note that the longitudinal response has been computed with an “artificial” width of 1 MeV. The uncorrelated Hartree response displays a large amount of

dipole strength around 8 MeV of excitation energy. This strength is concentrated in the “ $1-\hbar\omega$ ” region where many particle-hole excitations can be made. Yet most of the strength is spurious, as revealed by the large amount being shifted to zero excitation energy in the RPA response. What remains is an almost imperceptible fragment located at $E = 24.4$ MeV; and nothing else. The small fragment is displayed more clearly along with the experimental value (shown as a filled circle) on the inset of the figure. This result is a testimony to the power of consistency. By demanding that the residual particle-hole interaction be identical to the interaction in the ground state, and by properly including the mixing between positive- and negative-energy states, all spurious strength gets cleanly separated from the physical response.

A comparison between three different relativistic models—all of them constrained to reproduce bulk properties of nuclear matter at saturation as well as the root-mean-square charge radius of ^{40}Ca —is displayed in Fig. 2. Note that the three models employed here have been defined in Ref. [6] as L2 ($K = 547$ MeV), NLB ($K = 421$ MeV), and NLC ($K = 224$ MeV) [see also Table I]. As expected, the energy of the dipole resonance scales with the compressibility of the model. Clearly, models with a large compression modulus—such as L2 and NLB—produce isoscalar dipole strength at values that are too large to be consistent with experiment [13,14]. These results have also been tabulated in Table II. Because of the heroic efforts by experimentalists in separating the isoscalar dipole mode from the high-energy octupole resonance (HEOR), we also include a comparison between our results and their experimental findings in Table II. Although not necessarily a compressional mode, our results for the HEOR follow similar trends to those observed for the giant dipole resonance.

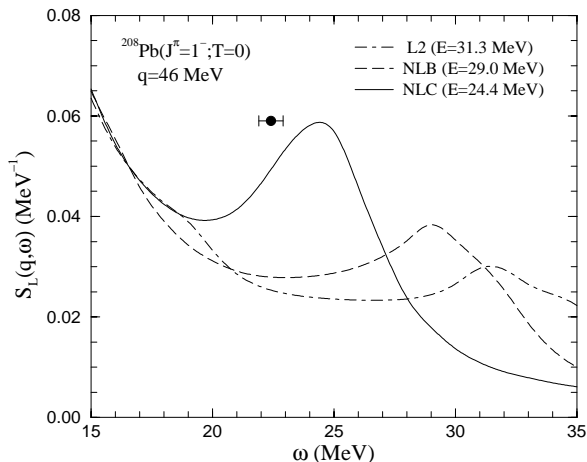


FIG. 2. Distribution of isoscalar dipole strength in ^{208}Pb at a momentum transfer of $q = 46$ MeV for three different models. The experimental value is from Ref. [14].

We conclude the presentation of our results by dis-

playing the distribution of strength for the quintessential compressional mode: the isoscalar giant monopole resonance (GMR). First discovered in α -scattering experiments from ^{208}Pb [15], and recently measured with higher accuracy at an excitation energy of $E = 14.2 \pm 0.1$ [16], the GMR places important constraints on theoretical models of nuclear matter. Indeed, the first measurement of the GMR—in conjunction with a simple analysis based on the liquid-drop model—suggested a compression modulus of about $K = 200$ MeV, a value considerably lower than the predictions of density-dependent Skyrme models at the time. Our calculations for the monopole strength in ^{208}Pb are displayed along with the experimental value in Fig. 3. We find good agreement with empirical formulas that suggest that the position of the GMR should scale as the square root of the compressibility. Indeed, we compute GMR energies in the ratio of 1:1.38:1.53, while the square root of the nuclear-matter compressibilities are in the ratio of 1:1.37:1.56. Moreover, these results help to reinforce our earlier claim that relativistic models of nuclear structure having compression moduli well above $K \approx 200$ MeV will be in conflict with experiment.

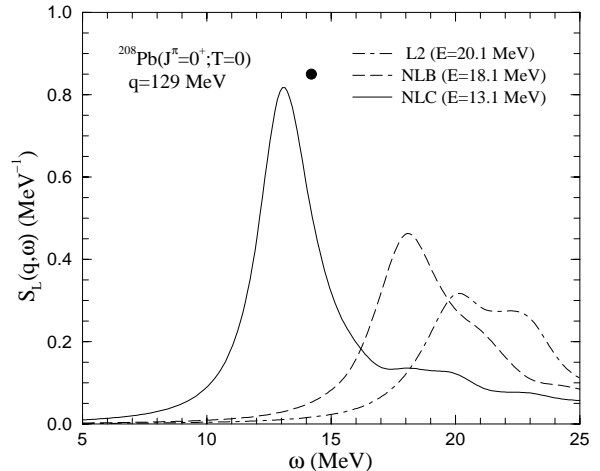


FIG. 3. Distribution of isoscalar monopole strength in ^{208}Pb at a momentum transfer of $q = 129$ MeV for three different models. The experimental value is from Ref. [16].

In summary, we have computed the distribution of strength for the isoscalar monopole, dipole, and high-energy octupole resonances in ^{208}Pb using a relativistic random-phase approximation to three different parameterizations of the Walecka model with scalar self-interactions. We placed particular emphasis on the role of consistency. That is, we demanded that the residual particle-hole interaction used in the RPA be identical to the interaction used to generate the mean-field ground state. Moreover, we have used a nonspectral approach that automatically included the mixing between positive- and negative-energy states to compute the longitudinal response. Enforcing these constraints—and little else—was sufficient for separating the spurious $J^\pi = 1^-; T = 0$

state. In contrast to recent relativistic calculations [8]—as well as nonrelativistic ones [17,18]—we see no need for imposing additional constraints to “partially” remove the spurious contamination. These approaches attempt to remove all spurious strength by defining an effective dipole operator of the form $M_{10}(\mathbf{r}) = (r^3 - \eta r)Y_{10}(\hat{\mathbf{r}})$; here η plays the role of a Lagrange multiplier and is determined to be $\eta = 5\langle r^2 \rangle / 3$ from translational invariance. More significantly, such a transition operator neglects the all important momentum dependence of the excitation. Indeed, it was only at small momentum transfers—just as in the experiment [14]—that we observed a single physical fragment concentrated around 24 MeV of excitation. As the momentum transfer increased, we uncovered additional dipole strength around 8 MeV. Yet this trend, namely, a sizable fraction of dipole strength at low energies and a giant resonance peak, is all that was reported in those recent publications [8,18]. We do not regard this behavior as “*a large discrepancy between theory and experiment*”. Rather, we attribute this trend to the simplified—momentum-independent—choice of dipole operator adopted in those calculations.

We have also computed the distribution of strength for the giant monopole resonance. As in the case of ISGDR we have used the same exact operator—the isoscalar vector density—to compute the monopole component of the longitudinal response. Indeed, monopole, dipole, and octupole strength were all obtained from simply isolating the relevant J^π -channel from the longitudinal response. For the GMR we have found good agreement with a recent relativistic calculation [7]. Good agreement has also been obtained with semi-empirical formulas that suggest that the position of the GMR should scale as the square root of the compressibility. Depending on the relativistic parameterization adopted, monopole strength was found between 13 and 20 MeV of excitation.

Lastly, we venture into the neutron-star domain. The non-linear parameterization NLC gives a rather satisfactory description of the various compressional modes. Although by no means perfect, this agreement suggests that the compression modulus of nuclear matter can not differ too much from the value predicted by this model ($K = 224$ MeV). When this parameter set is used to compute the equation of state for neutron matter—and is then combined with the Tolman-Oppenheimer-Volkoff equation—one obtains an upper limit for the mass of a neutron star of $M = 2.8M_\odot$. Given the recent compilation of 21 neutron-star masses by Thorsett and Chakrabarty [19], where they show that the measurements are consistent with a remarkably narrow mass distribution $M = (1.35 \pm 0.04)M_\odot$ [19], the fascinating possibility that neutron stars harbor novel and exotic states of matter becomes almost a reality.

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TABLE I. Various relativistic parameter sets [6]. The scalar mass and κ are in MeV.

Set	g_s^2	g_v^2	g_ρ^2	m_s	κ	λ
L2	109.63	190.43	65.23	520	0	0
NLB	94.01	158.48	73.00	510	800	10
NLC	95.11	148.93	74.99	501	5000	-200

TABLE II. Energies for various isoscalar resonances in three different relativistic models. All excitation energies are given in MeV.

Model	GMR	ISGDR	HEOR
L2	20.1	31.3	25.1
NLB	18.1	29.0	23.4
NLC	13.1	24.4	21.9
Exp.	14.2 ± 0.1 [16]	22.4 ± 0.5 [14]	19.7 ± 0.5 [14]

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